1 Resolving power definition

The job of a grating is to take light that is propagating along one direction, $\hat{\mathbf{k}}_i$, and redirect it along another wavelength-dependent direction

$$\hat{\mathbf{k}}_f = \hat{\mathbf{k}}_i - \lambda \mathbf{G}\,, \tag{1}$$

where λ is the wavelength of the light, and $\mathbf{G}$ is a fixed scattering vector that is a property of the grating. If the spatial frequency or wavenumber of the grating is ν,

$$\mathbf{G} = m\nu\hat{\mathbf{x}}\,, \tag{2}$$

where m is the diffraction order, and $\hat{\mathbf{x}}$ is the grating's periodic direction. Equation (1) is usually called the *grating equation*.

For any real grating, the unique $\mathbf{G}$ is smeared out into a distribution of scattering vectors, $\mathbf{Q}$, so that Eq. (1) becomes

$$\left\langle \hat{\mathbf{k}}_f \right\rangle = \hat{\mathbf{k}}_i - \lambda \left\langle \mathbf{Q} \right\rangle\,, \tag{3}$$

where $\langle \cdots \rangle$ is an average over the distribution, which is centered near $\mathbf{G}$. As a result, the grating does not direct each λ along a unique $\hat{\mathbf{k}}_f$. If the width of the distribution is ΔQ, we can no longer resolve wavelengths that are separated by less than

$$\Delta\lambda = \lambda \frac{\Delta Q}{G}\,. \tag{4}$$

The *resolving power* of a grating is the dimensionless ratio

$$\frac{G}{\Delta Q} = \frac{\lambda}{\Delta\lambda}\,. \tag{5}$$

A grating's resolving power is a key measure of its performance because it is an intrinsic property of the grating. No matter how the grating is used, it cannot avoid mixing together wavelengths that are separated by less than $\Delta\lambda$, and therefore the resolving power of any optical system which includes a grating can never exceed the intrinsic resolving power of that grating.

2 Measuring resolving power

The intrinsic resolving power of an ideal, diffraction-limited grating is

$$\frac{\lambda}{\Delta\lambda} = GL\,, \tag{6}$$

where L is the length of the grating. For a non-ideal grating, whose constituent grooves are displaced from their ideal positions, the resolving power of the grating becomes a complicated function of G, and we must determine the shape of the scattering-vector distribution empirically.

The scattering-vector distribution can be measured directly using a diffractometer, since the intensity of radiation along a particular $\hat{\mathbf{k}}_f$ is given by the amplitude of

$$\mathbf{Q} = \lambda^{-1}\left(\hat{\mathbf{k}}_i - \hat{\mathbf{k}}_f\right) \tag{7}$$

in the distribution. However, a direct measurement requires a diffractometer whose resolving power is *greater* than the resolving power of the grating under test. Often, a better approach is to instead measure the imperfect grating structure, and then *calculate* the scattering-vector distribution using the measured structure.

To measure the grating structure, we employ a Fizeau interferometer. The method is reported elsewhere [1, 3]. Briefly, visible-light interferometry is used to record aberrations across diffracted wavefronts from the grating, which can be converted directly to imperfections in the grating structure. Concretely, we obtain the number of grooves or periods, $N(x)$, between position x on the surface, and a reference position.

The *groove-number function*, $N(x)$, is a fixed property of the grating which only needs to be measured once. Then the scattering-vector distribution amplitude, $S(Q)$, near the mth mode is

$$S(Q) \propto \left| \int \exp[iQx] \exp[-imN(x)] \, \mathrm{d}x \right|, \tag{8}$$

where the integration is over the length of the grating, and $|\cdots|$ is the scalar norm [2].

References

[1] Samuel Gleason, Jonathan Manton, Janet Sheung, Taylor Byrum, Cody Jensen, Lingyun Jiang, Joseph Dvorak, Ignace Jarrige, and Peter Abbamonte. Intrinsic resolving power of XUV diffraction gratings measured with Fizeau interferometry. In Lahsen Assoufid, Haruhiko Ohashi, and Anand Krishna Asundi, editors, *Advances in Metrology for X-Ray and EUV Optics VII*, volume 10385, page 1038506. International Society for Optics and Photonics, SPIE, 2017.

[2] Malcolm R. Howells. X-ray data booklet. `https://xdb.lbl.gov/`, 1985. Section 4.3.

[3] George W Stroke. Diffraction gratings. *Handbuch der Physik*, 29:426–754, 1967.